

Adrian Harrison

# INTRODUCTION TO LOGARITHMS

## PRE-CALCULUS

1. Only positive numbers have logarithms.
2.  $\log_a 1 = 0$
3.  $\log_a a = 1$
4.  $y = \log_a x \Leftrightarrow x = a^y$
5.  $a^{\log_a x} = x$
6.  $\log_a (x \cdot y) = \log_a x + \log_a y$
7.  $\log_a \frac{x}{y} = \log_a x - \log_a y$
8.  $\log_a x^n = n \cdot \log_a x$
9.  $\log_a x = \frac{\log_b x}{\log_b a}$  changing the base of a logarithm
10.  $\log_a x = \frac{1}{\log_x a}$

**EASY MATHEMATICS**



**By Adrian Harrison**

# **INTRODUCTION TO LOGARITHM**

Copyright © 2020

All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission of the publisher, except in the case of brief quotations embodied in critical reviews and certain other noncommercial uses permitted by copyright law. For permission requests, write to the publisher using address below.

[delightfulbook@gmail.com](mailto:delightfulbook@gmail.com)

© 2020

# Contents

LOGARITHM

Definition:

PROPERTIES

INVERSE OF A LOGARITHM FUNCTION

TEST WITH SOLUTIONS

Test 1

Test 2

Test 3

Test 4

Test 5

# LOGARITHM

## Definition:

$$A(a) = 1$$

$$A(x) = \log_a b .$$

$$B(x) = - \log_a c .$$

## PROPERTIES

1. Only positive numbers have logarithms.

2.  $\log_a 1 = 0$

3.  $\log_a a = 1$

4.  $y = \log_a x \Leftrightarrow x = a^y$

5.  $a^{\log_a x} = x$

6.  $\log_a (x \cdot y) = \log_a x + \log_a y$

7.  $\log_a \frac{x}{y} = \log_a x - \log_a y$

8.  $\log_a x^n = n \cdot \log_a x$

9.  $\log_a x = \frac{\log_b x}{\log_b a}$  changing base of a logarithm

10.  $\log_a x = \frac{1}{\log_x a}$

11.  $\log_{a^n} x^m = \frac{m}{n} \log_a x$
12.  $\log_{10} x = \log x$
13.  $\log_e x = \ln x, e \equiv 2.7$
14.  $\text{Colog } a = -\log a$
15.  $\log_a b \cdot \log_b c \cdot \log_c d \cdot \log_d f = \log_a f$

**Example:**

$$A = \frac{2}{\log_{11} 385} + \frac{2}{\log_7 385} + \frac{2}{\log_5 385}$$

$$\Rightarrow A = ?$$

**Solution:**

$$A = 2 \left( \frac{1}{\frac{1}{\log_{385} 11}} + \frac{1}{\frac{1}{\log_{385} 7}} + \frac{1}{\frac{1}{\log_{385} 5}} \right)$$

$$A = 2(\log_{385} 11 + \log_{385} 7 + \log_{385} 5)$$

$$A = 2(\log_{385} 11 \cdot 7 \cdot 5) = 2 \log_{385} 385$$

$$= 2 \cdot 1$$

$$= 2$$

**Example:**

$$2^{\log x} = 3^{\log 2} \Rightarrow x = ?$$

**Solution:**

$$\begin{aligned}\log 2^{\log x} &= \log(3^{\log 2}) \\ \log x \cdot \log 2 &= \log 2 \cdot \log 3 \\ \log x &= \log 3 \Rightarrow x = 3\end{aligned}$$

**Example:**

$$x^{\ln x} - e^6 \cdot x = 0 \Rightarrow (SS) = ?$$

**Solution:**

$$\begin{aligned}x^{\ln x} - e^6 \cdot x &= 0 \Rightarrow x^{\ln x} = e^6 \cdot x \\ \ln(x^{\ln x}) &= \ln e^6 - 6 \\ \ln^2 x - \ln x - 6 &= 0 \\ (\ln x - 3) \cdot (\ln x + 2) &= 0 \\ \ln x - 3 &= 0 \quad \ln x = -2 \\ \ln x - 3 &= 0 \quad \ln x = -2 \\ x &= e^3 \quad x = e^{-2} \\ (SS) &= \left\{ \frac{1}{e^2} e^3 \right\}\end{aligned}$$

**Example:**

$$\begin{aligned}\log_2(x - 1) + \log_2(3x + 1) &= 6 \\ \Rightarrow (SS) &= ?\end{aligned}$$

**Solution:**

$$\begin{aligned}x - 1 > 0 \Rightarrow x > 1, \quad 3x + 1 > 0 \Rightarrow x > -\frac{1}{3} \\ x &> 1 \\ \log_2(x - 1) \cdot (3x + 1) &= 6 \\ (x - 1) \cdot (3x + 1) &= 2^6 \\ 3x^2 - 2x - 65 &= 0\end{aligned}$$

$$(3x + 13)(x - 5) = 0 \Rightarrow x = -\frac{13}{5}$$

$$(SS) = \{5\}$$

**Example:**

$$e^x - 12e^{-x} - 4 = 0 \Rightarrow (SS) = ?$$

**Solution:**

$$e^x - \frac{12}{e^x} - 4 = 0 \Rightarrow e^{2x} - 4e^x - 12 = 0$$

$$e^x = t \Rightarrow t^2 - 4t - 12 = 0$$

$$(t + 2) \cdot (t - 6) = 0$$

$$t + 2 = 0 \Rightarrow t = -2, t - 6 = 0 \Rightarrow t = 6$$

$$e^x = -2 \Rightarrow \zeta_1 = \emptyset$$

$$e^x = 6 \Rightarrow x = \ln 6$$

$$(SS) = \{\ln 6\}$$

**Example:**

$$\begin{cases} \log xy^3 = 3 \\ \log \frac{x^2}{y} = -8 \end{cases} \Rightarrow (x, y) = (?, ?)$$

**Solution:**

$$\begin{cases} \log x + 3 \log y = 3 \\ 2 \log x - \log y = -8 \end{cases} \Rightarrow \begin{cases} \log x + 3 \log y = 3 \\ 6 \log x - 3 \log y = -24 \end{cases}$$



$$\begin{aligned}
 7 \log x &= -21 \Rightarrow \log x = -3 \Rightarrow x = 10^{-3} \\
 -3 + 3 \log y &= 3 \Rightarrow \log y = 2 \Rightarrow y = 10^2 \\
 (x,y) &= (10^{-3}, 10^2)
 \end{aligned}$$

### INVERSE OF A LOGARITHM FUNCTION

$$f(x) = \log_a x \Leftrightarrow f^{-1}(x) = a^x$$

**Example:**

$$f(x) = \log_5 (3x - 2) \Rightarrow f^{-1}(2) = ?$$

**Solution:**

$$f(x) = \log_5 (3x - 2)$$

$$\Rightarrow y = \log_5 (3x - 2)$$

$$5^y = 3x - 2$$

$$\Rightarrow x = \frac{5^y + 2}{3}$$

$$f^{-1}(x) = \frac{5^x + 2}{3}$$

$$\Rightarrow f^{-1}(2) = \frac{5^2 + 2}{3} = \frac{27}{3} = 9$$

$$\Rightarrow f^{-1}(2) = 9$$

**Example:**

$$f(x) = 2^{5x-3} - 28 \Rightarrow f^{-1}(100) = ?$$

**Solution:**

$$y = 2^{5x - 3} - 28$$

$$2^{5x - 3} = y + 28$$

$$5x - 3 = \log_2(y + 28)$$

$$x = \frac{\log_2(y + 28) + 3}{5}$$

$$f^{-1}(x) = \frac{\log_2 \cdot 8(x + 28)}{5}$$

$$\begin{aligned} f^{-1}(100) &= \frac{\log_2 2^3 \cdot 128}{5} \\ &= \frac{\log_2 2^{10}}{5} = \frac{10}{5} = 2 \end{aligned}$$

# TEST WITH SOLUTIONS

1.  $\log 2 = m \Rightarrow \log 320 = ?$

- A)  $4m$       B)  $5m$       C)  $5m - 1$       D)  $5m$   
                      + 1  
                      E)  $m^5$

**Solution:**

$$\begin{aligned}\log 2 &= m \\ \log 320 &= \log (32 \cdot 10) = \log 32 + \log 10 \\ &= \log 2^5 + 1 \\ &= 5\log 2 + 1 \\ &= 5m + 1\end{aligned}$$

**Correct Answer - D**

2.  $\log 2 = m$  and  $\log 3 = n \Rightarrow \log 720 = ?$

- A)  $3n - 1$       B)  $2n + 1$       C)  $2m + 3m$   
 $+ 1$   
D)  $2n + 1$       E)  $m^5$

**Solution:**

$$\begin{aligned}\log 2 &= m, \log 3 = n \\ \log 720 &= \log(72 \cdot 10) = \log 72 + \log 10 \\ &= \log(9 \cdot 8) + 1 \\ &= \log 9 + \log 8 + 1 \\ &= \log 3^2 + \log 2^3 + 1 \\ &= 2\log 3 + 3\log 2 + 1 \\ &= 2n + 3m + 1\end{aligned}$$

**Correct Answer - C**

3.  $\log_2 3 \cdot \log_3 5 \cdot \log_5 9 \cdot \log_9 16 = ?$

- A) 1      B) 2      C) 3      D) 4      E) 5

**Solution:**

$$\begin{aligned} & \log_2 3 \cdot \log_3 5 \cdot \log_5 9 \cdot \log_9 16 \\ &= \frac{\log 3}{\log 2} \cdot \frac{\log 5}{\log 3} \cdot \frac{\log 3^2}{\log 5} \cdot \frac{\log 2^4}{\log 3^2} \\ &= \frac{\log 3}{\log 2} \cdot \frac{\log 5}{\log 3} \cdot \frac{2\log 3}{\log 5} \cdot \frac{4\log 2}{2\log 3} = 4 \end{aligned}$$

**Correct Answer - D**

4.  $\log_3 5 = a \Rightarrow \log_5 9 = ?$

- A) a      B) 2a      C)  $\frac{2}{a}$       D) - a      E)  $-\frac{a}{2}$

**Solution:**

$$\begin{aligned} & \log_3 5 = a \\ & \log_5 9 = \frac{1}{\log_9 5} = \frac{1}{\log_{3^2} 5} = \frac{2}{\log_3 5} = \frac{2}{a} \end{aligned}$$

**Correct Answer - C**

5.  $\log_2 (x - 5) = 4 \Rightarrow x = ?$

- A) 21      B) 16      C) 8      D) - 8      E)  
- 16

**Solution:**

$$\begin{aligned} \log_2 (x - 5) = 4 & \Rightarrow x - 5 = 2^4 \\ x - 5 &= 16 \end{aligned}$$

$$x = 16 + 5$$

$$x = 21$$

**Correct Answer - A**

$$6. \log_3 x - \log_3 (x - 1) = 2 \Rightarrow x = ?$$

$$A) \frac{8}{9}$$

$$B) \frac{9}{8}$$

$$C) \frac{2}{3}$$

$$D) -\frac{2}{3}$$

$$E) -\frac{9}{10}$$

**Solution:**

$$\log_3 x - \log_3 (x - 1) = 2 \Rightarrow \log_3 \left( \frac{x}{x - 1} \right) = 2$$

$$\frac{x}{x - 1} = 3^2 \Rightarrow \frac{x}{x - 1} = 9$$

$$x = 9x - 9$$

$$x = \frac{9}{8}$$

**Correct Answer - B**

$$7. e^{2x} - 4e^x - 32 = 0 \Rightarrow x = ?$$

$$A) \ln 2 \quad B) 3 \ln 2 \quad C) \ln 6 \quad D) 2 \ln 6 \quad E) 4 \ln 6$$

**Solution:**

$$e^{2x} - 4e^x - 32 = 0$$

$$(e^x - 8) \cdot (e^x + 4) = 0$$

$$e^x - 8 = 0$$

$$e^x = 8$$

$$x = \ln 8$$

$$x = 3 \cdot \ln 2$$

**Correct Answer - B**

$$8. \log_6 2 = a \Rightarrow \log_6 9 = ?$$

- A)  $3a$       B)  $6 - 3a - 4$       C)  $-2a$       D)  $2a$   
E)  $2 - 2a$

**Solution:**

$$\log_6 2 = \frac{1}{\log_2 6} = \frac{1}{\log_2 3 + 1} = a$$

$$\log_2 3 = \frac{1}{a} - 1 = \frac{1 - a}{a} \Rightarrow \log_3 2 = \frac{a}{1 - a}$$

$$\log_6 9 = \frac{1}{\log_9 6} = \frac{2}{\log_3 6} = \frac{2}{\log_3 2 + 1}$$

$$= \frac{2}{\frac{a}{1 - a} + 1}$$

$$= \frac{2}{\frac{1}{1 - a}}$$

$$= 2 \cdot (1 - a) = 2 - 2a$$

**Correct Answer - E**

$$9. \log_2 (\log_3 x) = 3 \Rightarrow x = ?$$

- A)  $2^3$       B)  $2^6$       C)  $3^8$       D)  $3^6$       E)  $-3^4$

**Solution:**

$$\log_2 (\log_3 x) = 3 \Rightarrow \log_3 x = 2^3$$

$$\log_3 x = 8$$

$$x = 3^8$$

**Correct Answer - C**

10.  $\log 20 + 2 \log 2 - 3 \log 2 = ?$

- A) - 2      B) - 1      C) 0      D) 1      E) 2

**Solution:**

$$\begin{aligned} \log 20 + 2\log 2 - 3\log 2 &= \log 20 + \log 2^2 - \log 2^3 \\ &= \log \left( \frac{20 \cdot 4}{8} \right) \\ &= \log 10 \\ &= 1 \end{aligned}$$

**Correct Answer - D**

11.  $\log_3 x + \log_9 x = 5 \Rightarrow x = ?$

- A)  $\sqrt[3]{3}$     B)  $3 \sqrt[3]{9}$     C)  $3 \sqrt[3]{3}$     D)  $27 \sqrt{3}$     E)  $27 \sqrt[3]{3}$

**Solution:**

$$\begin{aligned} \log_3 x + \log_9 x &= \log_3 x + \frac{1}{2} \log_3 x \\ &= \log_3 x + \log_3 x^{\frac{1}{2}} \\ &= \log_3 (x \cdot x^{1/2}) \\ &= \log_3 x^{3/2} = 5 \\ x^{3/2} &= 3^5 \\ x &= 3^{10/3} \\ x &= 27 \sqrt[3]{3} \end{aligned}$$

**Correct Answer - E**

$$12. \log_2(x + 2) + \log_2(x - 2) = 3 \Rightarrow x = ?$$

A)  $-2\sqrt{3}$     B)  $\frac{\sqrt{3}}{2}$     C)  $\sqrt{3}$     D)  $2\sqrt{3}$     E)  $2\sqrt{3}$

**Solution:**

$$\log_2(x + 2) + \log_2(x - 2) = 3$$

$$\log_2[(x + 2) \cdot (x - 2)] = 3$$

$$\log_2(x^2 - 4) = 3$$

$$x^2 - 4 = 2^3$$

$$x^2 = 12$$

$$x = 2\sqrt{3}$$

**Correct Answer - D**

$$13. \log 3 = a, \log 4 = b \Rightarrow \log_5 36 = ?$$

A)  $2a + 4b$     B)  $\frac{5 - 2a}{b + 1}$     C)  $\frac{a + 2b}{b - a}$     D)  $\frac{2b + 4a}{2 - b}$

E)  $\frac{4a - 2b}{a - 2}$

**Solution:**

$$\log 3 = a$$

$$\log 4 = \log 2^2 = 2\log 2 = b$$

$$\log 2 = \frac{b}{2}$$



$$\begin{aligned}
 \log_5 36 &= \frac{\log 36}{\log 5} = \frac{2 \log 6}{\log \frac{10}{2}} \\
 &= \frac{2(\log 2 + \log 3)}{\log 10 - \log 2} \\
 &= \frac{2\left(\frac{b}{2} + a\right)}{1 - \frac{b}{2}} \\
 &= \frac{2(b + 2a)}{2 - b} \\
 &= \frac{2b + 4a}{2 - b}
 \end{aligned}$$

**Correct Answer - D**

$$14. \log x^2 + \log x^3 = 15 \Rightarrow x = ?$$

- A)  $10^3$       B)  $10^5$       C)  $6^{15}$       D)  $2^{15}$   
 E)  $3^{10}$

**Solution:**

$$\begin{aligned}
 \log x^2 + \log x^3 &= \log (x^2 \cdot x^3) = \log x^5 \\
 \log x^5 &= 15 \Rightarrow x^5 = 10^{15} \\
 x &= 10^3
 \end{aligned}$$

**Correct Answer - A**

$$15. 3^{\log 3^8} + 2^{\log 2^9} = 5^{\log 5^x} \Rightarrow x = ?$$

- A) 17      B) 16      C) 15      D) 14      E) 13

**Solution:**

$$\begin{cases} 3^{\log 3^8} = 8 \\ 2^{\log 2^9} = 9 \\ 5^{\log 5^x} = x \end{cases} \Rightarrow \begin{aligned} 8 + 9 &= x \\ 17 &= x \end{aligned}$$

**Correct Answer - A**

$$16. \ln \sqrt{x} + \ln \sqrt{x^3} = 1 \Rightarrow x = ?$$

- A)  $2e$       B)  $e^2$       C)  $\sqrt{e}$       D)  $\sqrt[3]{e}$       E)  $e$

**Solution:**

$$\begin{aligned} \ln \sqrt{x} + \ln \sqrt{x^3} &= \ln (\sqrt{x} \cdot \sqrt{x^3}) \\ &= \ln \sqrt{x^4} \\ &= \ln x^2 = 1 \\ x^2 &= e \\ x &= \sqrt{e} \end{aligned}$$

**Correct Answer - C**

$$17. 3^x + 3^{x+2} = 10 \Rightarrow x = ?$$

- A) 0      B)  $\frac{1}{2}$       C) 1      D)  $\frac{3}{2}$       E)  $\frac{5}{2}$

**Solution:**

$$\begin{aligned} 3^x + 3^{x+2} &= 3^x + 3^x \cdot 3^2 = 10 \\ 3^x(1 + 9) &= 10 \\ 3^x &= 1 \end{aligned}$$

$$x = 0$$

**Correct Answer - A**

$$18. \log 2 = 0.30103 \Rightarrow \log 125 = ?$$

- A) - 2.69897      B) 2.69897      C) 2.60206  
D) 2.09691      E) - 2.6991

**Solution:**

$$\log 2 = 0.30103$$

$$\begin{aligned} \log 125 &= \log \frac{1000}{8} = \log 10^3 - \log 2^3 \\ &= 3 - 3\log 2 \\ &= 3 - 3 \cdot (0.30103) \\ &= 2.09691 \end{aligned}$$

**Correct Answer - D**

$$19. \log x = 2.48135 \Rightarrow \operatorname{colog} x = ?$$

- A)  $\overline{3.51865}$       B)  $e^{2.48135}$       C)  $\frac{1}{2.48135}$   
D)  $\overline{2.48135}$       E)  $\overline{3.48135}$

**Solution:**

$$\log x = 2.48135$$

$$\begin{aligned} \operatorname{colog} x &= -\log x \\ &= -2.48135 \\ &= -2 - 0.48135 \\ &= -2 - 1 + 1 - 0.48135 \\ &= -3 + 0.51865 \\ &= \overline{3.51865} \end{aligned}$$

**Correct Answer - A**

$$20. \log_3(x^2 - 6x) > 3$$

What is the solution set for this inequality?

- A)  $\begin{matrix} x < 3 \\ x > 9 \end{matrix}$       B)  $\begin{matrix} x < -3 \\ x > 9 \end{matrix}$       C)  $\begin{matrix} x < -3 \\ x > -9 \end{matrix}$
- D)  $\begin{matrix} x > 3 \\ x < 9 \end{matrix}$       E)  $\begin{matrix} x > -3 \\ x < -9 \end{matrix}$

**Solution:**

$$\log_3(x^2 - 6x) > 3$$

$$x^2 - 6x > 3^3 \quad \text{and} \quad x^2 - 6x > 0$$

$$x^2 - 6x > 27 \quad \quad \quad x(x - 6) > 0$$

$$x^2 - 6x - 27 > 0 \quad \quad \quad x_1 = 0$$

$$(x - 9) \cdot (x + 3) > 0 \quad \quad \quad x_2 = 6$$

$$x_1 = 9$$

$$x_2 = -3$$

$$x < -3, x > 9$$

**Correct Answer - B**

$$1. \quad x > 1$$

$$(x - 1)^{(x + 3)} = 1$$

$$\Rightarrow \log_{x-1} 1 = ?$$

- A) - 1      B) - 2      C) - 3      D) - 4      E) - 5

**Solution:**

$$x - 1 = 1 \Rightarrow x = 2$$

(Base of a logarithm cannot be negative)

$$x = 2$$

$$\log_2 \frac{1}{2} = -\log_2 2$$
$$= -1$$

**Correct Answer - A**

$$2. \log_3 5 = x \Rightarrow \log_3 15 = ?$$

$$A) 2x + 2$$

$$B) 2x + 1$$

$$C) 2x - 1$$

$$D) x - 2$$

$$E) x + 1$$

**Solution:**

$$\log_3 5 = x$$

$$\log_3 5 = \log_3 (3 \cdot 5) = \log_3 3 + \log_3 5 = 1 + x$$

**Correct Answer - E**

$$3. \log_{10} 5 = x \Rightarrow 5^{1-x} = ?$$

$$A) 2^x$$

$$B) 2^{-x}$$

$$C) 2^{x-1}$$

$$D) 2^{1-x}$$

$$E) 2^{x+1}$$

**Solution:**

$$\log_{10} 5 = x \Rightarrow 5 = 10^x$$

$$5 = 5^x \cdot 2^x$$

$$\frac{5}{5^x} = 2^x$$

$$5^{1-x} = \frac{5}{5^x} = 2^x$$

**Correct Answer - A**

$$\begin{aligned} 4. \quad |AB| &= \log_2 8 \\ |BC| &= \log_2 4 \\ \Rightarrow \frac{|AC|}{|BC|} &= ? \end{aligned}$$

$$\text{A) } \frac{5}{2} \quad \text{B) } \frac{3}{2} \quad \text{C) } \frac{1}{2} \quad \text{D) } 2 \quad \text{E) } 4$$

**Solution:**

$$\begin{aligned} \frac{|AC|}{|BC|} &= \frac{\log_2 8 + \log_2 4}{\log_2 4} = \frac{\log_2 (8 \cdot 4)}{\log_2 2^2} \\ &= \frac{\log_2 2^5}{2} \\ &= \frac{5}{2} \end{aligned}$$

**Correct Answer - A**

$$5. \quad \log_a \frac{a}{b} = 4 \Rightarrow \log_a b = ?$$

$$\text{A) } -1 \quad \text{B) } -2 \quad \text{C) } -3 \quad \text{D) } -4 \quad \text{E) } -5$$

**Solution:**

$$\log_a \frac{a}{b} = 4$$

$$\log_a a - \log_a b = 4$$

$$1 - \log_a b = 4$$

$$\log_a b = -3$$

**Correct Answer - C**

$$6. \frac{1}{\log_4 16} + \frac{1}{\log_2 4} = ?$$

A) 1      B) 2      C) 3      D)  $\frac{1}{2}$       E)  $\frac{1}{3}$

**Solution:**

$$\begin{aligned} \frac{1}{\log_4 16} + \frac{1}{\log_2 4} &= \frac{1}{\log_{2^2} 2^4} + \frac{1}{\log_2 2^2} \\ &= \frac{1}{2} + \frac{1}{2} \\ &= 1 \end{aligned}$$

**Correct Answer - A**

$$7. \begin{cases} \log_a x = 30 \\ \log_b x = 70 \end{cases} \Rightarrow \log_{ab} x = ?$$

A) 15      B) 21      C) 28      D) 35      E) 50

**Solution:**

$$\log_{ab} x = \frac{1}{\log_x ab} = \frac{1}{\log_x a + \log_x b}$$

$$= \frac{1}{\frac{1}{30} + \frac{1}{70}} = \frac{1}{\frac{10}{210}} = 21$$

**Correct Answer - B**

8.  $x \in \mathbb{R}^+, x \neq 1$   
 $\log_3(3 \cdot \log_x(2x - 3)) = 1 \Rightarrow x = ?$

A) 1      B) 2      C) 3      D) 4      E) 5

**Solution:**

$$\log_3(3 \cdot \log_x(2x - 3)) = 1$$

$$3 \cdot \log_x(2x - 3) = 3^1 = 3$$

$$\log_x(2x - 3) = 1$$

$$2x - 3 = x$$

$$x = 3$$

**Correct Answer - C**

9.  $\begin{cases} \log 3 = x \\ \log 5 = y \\ \log 7 = z \end{cases} \Rightarrow \log \frac{225}{7} = ?$

A)  $x + y - z$

B)  $x + 2y - z$

C)  $2x + y - z$

D)  $2x + 2y - z$

E)  $2x + 2y + z$

**Solution:**

$$\log \frac{225}{7} = \log 225 - \log 7 = \log(3^2 \cdot 5^2) - \log 7$$



$$= 2 \cdot \log 3 + 2 \cdot \log 5 - \log 7 = 2x + 2y - z$$

**Correct Answer - D**

---

## Logarithm

---

### Test 1

1.  $\log_6 2 + \log_6 3 = ?$

- A) 2      B) 1      C) 0      D) - 1      E) - 2

2.  $\log_9 27 = ?$

- A)  $\frac{1}{3}$       B) 3      C) 6      D)  $\frac{2}{3}$       E)  $\frac{3}{2}$

3.  $y = \log_7^{\frac{1}{x}}, x = 7^5 \Rightarrow y = ?$

- A) 1      B) 0      C) - 5      D) - 7      E) - 49

4.  $\log_3 5 = a \Rightarrow \log_5 15 = ?$

- A)  $a + 1$       B)  $a - 1$       C)  $1 + \frac{1}{a}$   
D)  $\frac{a - 1}{a}$       E)  $\frac{1}{a}$

5.  $\log_{\sqrt{2}}^1 8 = ?$

A) 0      B) - 2      C) - 4      D) - 6      E) - 8

$$6. \log_{\sqrt{8}} b = \frac{10}{3} \Rightarrow b = ?$$

A) 8      B) 16      C) 32      D) 64      E) 128

$$7. \log_{\frac{1}{x}} 4 = -2 \Rightarrow x = ?$$

A) 1      B) 2      C) 3      D) 4      E) 5

$$8. \log_x 4 = -\frac{1}{3} \Rightarrow x = ?$$

A)  $\frac{1}{4}$       B)  $\frac{1}{16}$       C)  $\frac{1}{24}$       D)  $\frac{1}{64}$       E) - 1

$$9. \begin{cases} \log_3 2 = a \\ \log_3 5 = b \end{cases} \Rightarrow \log 30 = ?$$

A)  $1 + \frac{1}{a+b}$       B)  $1 - \frac{1}{a+b}$       C)  $\frac{a}{b+1}$

D)  $a - b + 1$                       E)  $1 - \frac{a}{b}$

10.  $\log_3[\log_2(\log_4(x - 1))] = 0 \Rightarrow x = ?$

- A) 17              B) 18              C) 19              D) 20              E) 21

11.  $(\log_x 8)^{\log_5 125} = 27 \Rightarrow x = ?$

- A) 5              B) 4              C) 3              D) 2              E) 1

12.  $\log_2 m = \log_{\frac{1}{2}} n, m + n = 5$

$\Rightarrow m^2 + n^2 = ?$

- A) 27              B) 26              C) 25              D) 24              E) 23

13.  $\begin{cases} \log(xy) = 2 \\ \log\left(\frac{x}{y}\right) = -2 \end{cases} \Rightarrow y = ?$

- A) 1              B) 10              C) 100              D) 1000              E)  $\frac{1}{10}$

14.  $\log 2 = a \Rightarrow \log 25 = ?$

A)  $1 - a$

B)  $2 - a$

C)  $1 + a$

D)  $2 + a$

E)  $2 - 2a$

15.  $\log_{\sqrt{2}} 16 + \log_3 \sqrt{27} + \log_{25} 5 = ?$

A) 10

B) 9

C) 8

D) 7

E) 6

16.  $\log_7(\log_2 16) = \frac{1}{\log_x 49} \Rightarrow x = ?$

A) 64

B) 16

C) 8

D) 4

E) 2

17.  $\log_3 12 = a \Rightarrow \log_3 18 = ?$

A)  $\frac{a+1}{2}$

B)  $\frac{a+2}{2}$

C)  $\frac{a+3}{2}$

D)  $\frac{a-1}{2}$

E)  $\frac{a-2}{2}$

18.  $\log_3 a = \log_{\frac{1}{81}} b \Rightarrow \log_a b = ?$

A) -4

B)  $-\frac{1}{2}$

C)  $-\frac{1}{3}$

D)  $-\frac{1}{4}$

E)  $-\frac{1}{6}$

19.  $7^{\log_3 x} = 49 \Rightarrow x = ?$

- A) 3      B) 6      C) 7      D) 8      E) 9

20.  $\log_3 2 \cdot \log_8 125 \cdot \log_{25} 81 = ?$

- A) 2      B) 3      C) 4      D) 5      E) 6

21.  $\frac{(\log_2 20)^2 - (\log_2 5)^2}{\log_2 10} = ?$

- A) 6      B) 5      C) 4      D) 3      E) 2

22.  $\log_2(\log_{10} x) = 3 \Rightarrow x = ?$

- A)  $10^4$       B)  $10^6$       C)  $10^8$   
D)  $10^9$       E)  $10^{12}$

23.  $3^n = a, \log_a 81^2 = n^2 \Rightarrow n = ?$

- A) -1      B) 0      C) 1      D) 2      E) 3

$$24. \log_a 2 + \log_a 4 + \log_a 8 = 24 \Rightarrow a = ?$$

- A) 4      B)  $2^{\frac{1}{4}}$       C)  $\sqrt{2}$       D)  $\sqrt[3]{2}$       E)

$$25. (\log_{a-1} 9)^{\log_2 18} = 16 \Rightarrow a = ?$$

- A) 1      B) 3      C) 4      D) 5      E) 6

Answers					
1. B	2. E	3. C	4. C	5. D	6. C
7. B	8. D	9. A	10. A	11. D	12. E
13. C	14. E	15. A	16. B	17. C	18. A
19. E	20. A	21. C	22. C	23. D	24. E
25. C					

---

## Logarithm

---

### Test 2

1.  $\log_3 4 = x \Rightarrow \log_3 162 = ?$

A)  $\frac{x-8}{2} + 4$       B)  $\frac{x+8}{2}$       C)  $x$   
 D)  $x-4$

E)  $\frac{x-4}{2}$

2.  $\frac{1}{\log_2 18} + \frac{1}{\log_6 18} + \frac{1}{\log_{27} 18} = ?$

A) 2      B) 3      C) 4      D) 5      E) 6

3.  $a = \log_4 5$ ,  $b = \log_{\frac{1}{5}} 4$ ,  $c = \log_5 4 \Rightarrow ? < ? < ?$

A)  $a < b < c$       B)  $c < b < a$       C)  $a < c < b$   
 D)  $b < c < a$       E)  $b < a < c$

4.  $\log_5 a - \log_5 b = 2 \Rightarrow \frac{10b-a}{5b} = ?$



- A) - 3                      B) - 4                      C) - 5                      D) - 6  
E) - 7

5.  $\log_3 63 = x, \log_7 81 = y \Rightarrow y = ?$

- A)  $\frac{4}{x}$                       B)  $\frac{4}{x+1}$                       C)  $\frac{4}{x-1}$                       D)  $\frac{4}{x-2}$   
E)  $\frac{4}{x+2}$

6.  $125^{\log_5 2} + \log_5 0.008 = ?$

- A) 7                      B) 6                      C) 5                      D) 4                      E) 3

7.  $\log x = b - \log a \Rightarrow x = ?$

- A)  $a \cdot b 10$                       B)  $10 a \cdot b$                       C)  $109 \cdot b$   
D)  $\frac{10^b}{a}$                       E)  $\frac{a \cdot b}{10}$

8.  $\begin{cases} \log 2 = a \\ \log 3 = b \end{cases} \Rightarrow \log 72 = ?$

- A)  $3a$                       B)  $a + b$                       C)  $3b$

D)  $3a + 2b$       E)  $2a + 2b$

9.  $\log 4 \cdot \log_4 9 \cdot \log_3 e = ?$

A) 1      B) 2      C) 4      D)  $\ln 5$       E)  $\frac{2}{\ln 10}$

10.  $\log 40 = x \Rightarrow \log 25 = ?$

A)  $3 - 2x$       B)  $2 - x$       C)  $3 - 4x$   
D)  $1 - x$

E)  $3 - x$

11.  $\log(2x + 4) - \log(x - 2) = 1 \Rightarrow x = ?$

A) 7      B) 6      C) 5      D) 4  
E) 3

12.  $2\sqrt{\ln x} - \ln \sqrt{x} = 0 \Rightarrow x = ?$

A)  $\{1, e^4\}$       B)  $\{e^4, e^{16}\}$       C)  $\{1, e^{16}\}$       D)  $\{2, e^4\}$

E)  $\{2, e^{16}\}$

13.  $\log_x 3 > \log_x(4 - x) \Rightarrow x \in ?$

A)  $(3, +\infty) - \{1\}$       B)  $(0,4) - \{1\}$       C)  $(0,3)$

D)  $(3,4)$       E)  $(4, +\infty)$

14.  $\log 2 = a, \log 3 = b$  and  $\log 7 = c \Rightarrow \log 420 = ?$

A)  $a + b + c - b + c - 1$       B)  $a + b + c + 1$       C)  $a$

D)  $a \cdot b \cdot c + 12$       E)  $a \cdot b \cdot c - 1$

15.  $\log_{15} 3 = a \Rightarrow \log_5 15 = ?$

A)  $a - 1$       B)  $a + 1$       C)  $3a$       D)  $\frac{1}{a + 1}$

E)  $\frac{1}{1 - a}$

16.  $\log_3 x + 5 \log_x 3 = 6 \Rightarrow x = ?$

A)  $\{3, 243\}$       B)  $\{3, 8\}$       C)  $\left\{\frac{1}{3}, \frac{1}{81}\right\}$

$$D) \left\{ \frac{1}{243}, \frac{1}{3} \right\} \quad E) \{27, 81\}$$

$$17. \quad \log_3 a - \log_{\frac{1}{3}} b = 3, \log_4 (a + b) = 2$$

$$\Rightarrow a^2 + b^2 = ?$$

$$A) 54 \quad B) 148 \quad C) 202 \quad D) 256 \quad E) 310$$

$$18. \quad \log_4 [\log_5 (\ln x)] = 0 \Rightarrow x = ?$$

$$A) 0 \quad B) 1 \quad C) e^3 \quad D) e^4 \quad E) e^5$$

$$19. \quad \log_5 (x + y) + \log_5 (x - y) = 2$$

$$x + y = 25 \Rightarrow x^2 + y^2 = ?$$

$$A) 83 \quad B) 125 \quad C) 193 \quad D) 313 \quad E) 625$$

$$20. \quad \log_{16} a + \log_4 a - \log_2 a = 0.5 \Rightarrow a = ?$$

$$A) \frac{1}{4} \quad B) \frac{1}{2} \quad C) \frac{2}{3} \quad D) 2 \quad E) 3$$

$$21. \quad \log_{10} (\log_8 x) + \log_{10} (\log_x 8) = ?$$

A) 0      B) 1      C) x      D) 8      E) 10

22.  $\begin{cases} \log_2 3 = a \\ \log_2 5 = b \end{cases} \Rightarrow \log 60 = ?$

A)  $\frac{a + b + 1}{b - 1}$       B)  $\frac{a + b}{1 + b}$       C)  $\frac{2 + a + b}{1 + b}$   
D)  $\frac{2 + a + b}{2 + b}$       E)  $\frac{a + b - 2}{1 + b}$

23.  $\log_5(-x) + \log_5(4 - x) = \log_5 12 \Rightarrow x = ?$

A) - 1      B) - 2      C) - 3      D)  
- 4  
E) - 5

24.  $\log_3(26!) = x \Rightarrow \log_3(27!) = ?$

A) 3x      B) 3 + x      C) 3 - x      D)  
) 2 + x  
E) 2

Answers					
1. B	2. A	3. D	4. A	5. D	6. C
7. D	8. D	9. B	10.	11.	12.

			E	E	C
13. B	14. B	15. E	16. A	17. C	18. E
19. D	20. A	21. A	22. C	23. B	24. B

---

## Logarithm

---

### Test 3

1.  $\log_2 3 = x \Rightarrow \log_9 2 = ?$

A)  $\frac{1}{2x}$       B)  $\frac{x}{2}$       C)  $\frac{x+1}{2}$

D)  $\frac{1}{x+2}$       E)  $\frac{2}{x+1}$

2.  $\log_a 9 = 6 \Rightarrow \log_{27} a = ?$

A)  $\frac{1}{9}$       B)  $\frac{1}{6}$       C)  $\frac{1}{4}$       D) 2      E) 5

3.  $\log_4 [\log_3 (\ln x)] = 0 \Rightarrow x = ?$

A) 12      B)  $e^{e^3}$       C) 64      D)  $e^2$       E)

4.  $\log_a b = 6 \Rightarrow \log_a bc + \log_a \frac{b}{c} = ?$

A) 15      B) 14      C) 13      D) 12      E)  
 ) 11

$$5. a^{\log a^6} + b^{\log b^{\frac{x}{5}}} = 9 \Rightarrow x = ?$$

- A) 15      B) 12      C) 10      D) 9      E) 6

$$6. \frac{1}{\log_9 3} + \log_3 x = 5 \Rightarrow x = ?$$

- A) 1      B) 3      C) 6      D) 9      E) 27

$$7. \frac{1}{\log_4 2} + \frac{1}{\log_8 2} + \frac{1}{\log_{16} 2} = ?$$

- A) 2      B) 3      C) 4      D) 7      E) 9

$$8. \log_4 8 \cdot \log_8 32 = ?$$

- A)  $\frac{5}{2}$       B)  $\frac{5}{3}$       C)  $\frac{3}{2}$       D) 1      E)  $\frac{1}{2}$

$$9. \log_x 3 + \log_9 x = \frac{3}{2} \Rightarrow \log(x^2 + 1) = ?$$

- A)  $\log 5$       B) 1      C) 2      D) 3      E)  $\log 17$

$$10. 100^{\log x} = x^2 - 2x + 4 \Rightarrow x = ?$$



A) 1      B) 2      C) 3      D) 4      E) 5

$$11. \log_3 16 \cdot \log_2 \frac{1}{27} = x \Rightarrow x = ?$$

A) - 12      B) - 6      C)  $\frac{2}{3}$       D) 6  
E) 12

$$12. \log_2 \left( \frac{1}{16} \right) = x \Rightarrow x = ?$$

A)  $\frac{1}{2}$       B)  $\frac{1}{4}$       C)  $\frac{1}{8}$   
D) - 2      E) - 4

$$13. \log \sqrt{125} \cdot \ln 10 \cdot \log_5 e = ?$$

A) 1      B)  $\frac{e}{10}$       C) e      D)  $\frac{3}{2}$       E)  $\frac{5}{3}$

$$14. \begin{cases} \log_b^{1/a} = 2 \\ \log_c b = 3 \end{cases} \Rightarrow \log_{\frac{1}{c}} a = ?$$

A)  $-\frac{1}{6}$       B)  $\frac{1}{6}$       C)  $\frac{1}{2}$       D)  $\frac{3}{2}$       E) 6

15.  $3 + \log_5 10 - \log_5 50 = ?$

A) -1      B) 0      C) 1      D) 2      E) 3

16.  $x = 27 \Rightarrow y = ?$

A) 3      B) 6      C) 9      D) 12      E) 36

17.  $3^{2 + \ln x} + 3^{\ln x} = 270 \Rightarrow x = ?$

A)  $e$       B)  $e^2$       C)  $e^3$       D)  $e^4$       E)  $e^5$

18.  $1 + \ln(e - x) = \ln(x + 3) \Rightarrow x = ?$

A)  $\frac{e + 3}{e - 1}$       B)  $\frac{e^2 - 1}{e + 3}$       C)  $\frac{e^2 - 3}{e + 1}$

D)  $\frac{e - 1}{e^2 + 3}$       E)  $\frac{e - 1}{e - 3}$

19.  $f(x) = \log_3(2x - m)$ ,  $f^{-1}(2) = 3 \Rightarrow m = ?$

- A) - 9                      B) - 6                      C) - 3                      D) 2  
E) 4

20.  $\log_{\frac{1}{2}}(x - 2) \geq 0 \Rightarrow (SS) = ?$

- A) [2,3)   B) (2,4)   C) (3,∞)   D) [2,3]   E) (2,3]

21.  $\log_3 5 = a \Rightarrow \log_{81} 15 = ?$

- A)  $\frac{a + 1}{4}$                       B)  $\frac{a - 1}{2}$                       C)  $\frac{a + 3}{5}$   
D)  $\frac{2a + 3}{2}$                       E)  $\frac{a + 3}{6}$

22.  $x > 0$ ,  $\log_2[\log_3(x^2 + 17)] = 2 \Rightarrow x = ?$

- A) 12                      B) 10                      C) 8                      D) 6                      E) 4

23.  $\begin{cases} \ln(xy) = 3 \\ \ln x - \ln y = 1 \end{cases} \Rightarrow x = ?$

- A) 1                      B) 2                      C) e                      D)  $e^2$                       E)  $e^3$

24.  $\log_5 (x - 2) + \log_5 (x + 2) = 1 \Rightarrow (SS) = ?$

- A)  $\{4\}$                       B)  $\{-3\}$                       C)  $\{3\}$   
 D)  $\{3, -3\}$                       E)  $\{-3, 5\}$

25.  $f(x) = 3 + 2 \cdot \log_{16}(3x - 2) \Rightarrow f^{-1}(3) = ?$

- A) 1                      B) 2                      C) 3                      D) 4                      E) 5

26.  $\log_{81} x + \log_{27} x = \log_3 x \Rightarrow \zeta K(SS) = ?$

- A)  $\emptyset$                       B)  $\{1\}$                       C)  $\left\{\frac{1}{3}, 1\right\}$   
 D)  $\left\{\frac{1}{3}\right\}$                       E)  $\{3\}$

Answers					
1. A	2. A	3. E	4. D	5. A	6. E
7. E	8. E	9. B	10. B	11. A	12. E
13. D	14. A	15. D	16. B	17. C	18. C
19. C	20. E	21. A	22. C	23. D	24. C
25.	26.				

A	B				
---	---	--	--	--	--

---

## Logarithm

---

### Test 4

1.  $\frac{1 + \log 90}{\log 30} = ?$

- A) 1      B) 2      C) 3      D) 4      E) 5

2.  $k \in \mathbb{Z}_+$  and  $0 < m < 1$

$\log(218672163.35) = k + m \Rightarrow k = ?$

- A) 6      B) 7      C) 8      D) 9      E) 10

3.  $5^n = a \Rightarrow \log_{25} a = ?$

- A)  $\frac{n}{10}$       B)  $5n$       C)  $2n$       D)  $\frac{n}{5}$       E)  $\frac{n}{2}$

4.  $\log \frac{x}{5} + 1 = \log x - \log(2 - x) \Rightarrow \sum x = ?$

- A)  $\frac{7}{5}$       B)  $\frac{5}{3}$       C)  $\frac{5}{4}$       D)  $\frac{3}{2}$       E)  $\frac{4}{5}$

5.  $\log x = a, \log y = b \Rightarrow \operatorname{colog} \frac{x}{y} = ?$

- A)  $\frac{a}{b}$       B)  $a + b$       C)  $b \cdot a$       D)  $a - b$   
 E)  $b - a$

6.  $\log_{27} x + \log_9 x = \frac{5}{2} \Rightarrow \log_{81} x = ?$

- A)  $\frac{2}{3}$       B)  $\frac{3}{5}$       C)  $\frac{3}{4}$       D)  $\frac{4}{3}$       E)  $\frac{5}{3}$

7.  $\log_4 7 = a \Rightarrow \log_7 28 = ?$

- A)  $\frac{2}{a}$       B)  $\frac{a+1}{a}$       C)  $\frac{a-1}{4}$   
 D)  $\frac{a+1}{4}$       E)  $\frac{2a+1}{2}$

8.  $\log 2 = a$

$\log 3 = b$

$\Rightarrow \log_5 18 = ?$

- A)  $\frac{a+b}{a-b}$       B)  $\frac{a(a+b)}{a-b}$       C)  $\frac{a+2b}{1-a}$

$$D) \frac{a(b+2a)}{b(1-a)} \quad E) \frac{b(a+2b)}{a(1-b)}$$

$$9. \log_3 \sqrt[3]{a \sqrt[3]{a \sqrt[3]{a} \dots}} = 2 \Rightarrow a = ?$$

$$A) 9 \quad B) 27 \quad C) 81 \quad D) 243 \quad E) 729$$

$$10. \log_x y \cdot \log_y x^2 \cdot \log_3 \left( \frac{x-1}{3} \right) = 2 \Rightarrow x = ?$$

$$A) 6 \quad B) 7 \quad C) 8 \quad D) 9 \quad E) 10$$

$$11. x^2 - x \log a + \log b = 0 \Rightarrow (SS) = \{x_1, x_2\}$$

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{1}{3} \quad \text{what is the relation}$$

between a and b?

$$A) b = a^3 \quad B) 3a = b \quad C) a = b^2$$

$$D) \frac{a}{b} = 3$$

$$E) a \cdot b = 2$$

$$12. \log_3 x + \log_x 3 = 2 \Rightarrow (SS) = ?$$



A)  $\{3,4\}$       B)  $\left\{3,\frac{1}{3}\right\}$       C)  $\{3\}$       D)  $\{2\}$

E)  $\{2,3\}$

13.  $\log(5x + 10)^2 - \log(3x - 4)^2 = 2 \Rightarrow x_1 = a$   
 $\Rightarrow \log 4a = ?$

A) 8      B) 4      C) 2      D)  $\frac{1}{4}$       E)  $\frac{1}{2}$

14.  $a, b > 1$

$$\log_b(\log_a \sqrt[b]{a}) = \log_a x \Rightarrow x^{-1} = ?$$

A)  $a$       B)  $\frac{1}{a}$       C)  $b$       D)  $\frac{1}{b}$       E)  $a \cdot b$

15.  $\frac{2}{3} \log(x^2 - y^2)$

$$- \frac{1}{2} [\log(x - y) + \log(x + y)] = ?$$

A)  $\log \sqrt{x - y}$     B)  $\log \sqrt[3]{x^2 + y^2}$

C)  $\log \sqrt[6]{x^2 + y^2}$

$$D) \log \sqrt[6]{x^2 - y^2}$$

$$E) \log \sqrt[3]{x^2 - y^2}$$

$$16. \log_{(b+c)} a + \log_{(c-b)} a =$$

$$2 \cdot \log_{(c+b)} a \cdot \log_{(c-b)} a$$

What is the relation between a, b and c?

$$A) b^2 = a^2 + c^2$$

$$B) c^2 = b^2 + a^2$$

$$C) a^2 = b^2 + c^2$$

$$D) a^2 = 2b + 2c$$

$$E) a = b + c$$

$$17. \log_{\frac{1}{3}} (\sin x) = 2 \Rightarrow \cos x = ?$$

$$A) \frac{4\sqrt{5}}{9}$$

$$B) \frac{2\sqrt{5}}{3}$$

$$C) \frac{\sqrt{5}}{9}$$

$$D) \frac{\sqrt{5}}{3}$$

$$E) \frac{2\sqrt{5}}{5}$$

$$18. x \in \mathbb{Z}$$

$$\log_4(2x - 5) < \log_2 3 \Rightarrow \sum x = ?$$

$$A) 10$$

$$B) 14$$

$$C) 15$$

$$D) 18$$

$$E) 22$$

$$19. \quad x^{\log x} = \frac{x^3}{100} \Rightarrow x_1 \cdot x_2 = ?$$

A) 10      B) 100      C) 400      D) 900      E) 1000

$$20. \quad (\log_4 x)^2 - 7\log_4 x + 12 = 0 \Rightarrow \sum x = ?$$

A) 64      B) 128      C) 250      D) 256      E) 320

$$21. \quad \log x + \log(2x + 1) = 0 \Rightarrow x = ?$$

A)  $\frac{1}{2}$       B) 1      C) 2      D)  $\frac{3}{2}$       E) 3

$$22. \quad \ln x = a \Rightarrow \log x^2 = ?$$

A)  $2a \cdot \log e$       B)  $a \cdot \log 2e$

C)  $\frac{a}{2} \cdot \log e$

D)  $2 \cdot \ln a$

E)  $\frac{2 \ln a}{3}$

Answers					

1. B	2. C	3. E	4. D	5. E	6. C
7. B	8. C	9. C	10. E	11. A	12. C
13. E	14. A	15. D	16. B	17. A	18. D
19. E	20. E	21. A	22. A		

---

## Logarithm

---

### Test 5

1.  $\log \frac{x^3 y^2}{z^4} = ?$

A)  $\frac{7 \log x \cdot \log y}{4 \cdot \log z}$  B)  $\frac{3xy}{2z}$  C)  $\frac{\log x^3 - y^2}{\log z^4}$

D) 3

$\log x + y^2 - z^4$

E)  $3 \log x + 2 \log y - 4 \log z$

2.  $\log_2 5 = x$  and  $\log_5 2 = y$

what is the relation between x and y?

A)  $x - y = 1$  B)  $x \cdot y = 12$  C)  $x \cdot y = 1$

D)  $\frac{x}{y} = \frac{4}{3}$

E)  $x + y = 7$

3.  $\frac{1}{4} \log a - \frac{3}{4} \log b + \log c = ?$

A)  $\log \frac{\sqrt[4]{a \cdot c}}{\sqrt[4]{b^3}}$  B)  $\log \frac{\sqrt[4]{b^2}}{\sqrt[4]{a \cdot c}}$  C)  $\log \sqrt[4]{a \cdot c} - b$

$$D) \log \frac{a^4 b^3}{c}$$

$$E) \log \frac{a^4 b^3}{\sqrt[4]{ab^3}}$$

$$4. \log_5 3 = x \Rightarrow \log_{15} 5 = ?$$

$$A) \frac{1}{x}$$

$$B) \frac{1}{x^2}$$

$$C) \frac{x+1}{3}$$

$$D) \frac{1}{x+1}$$

$$E) 1$$

$$5. f(x) = \log_3(3x+2) \Rightarrow f^{-1}(x) = ?$$

$$A) \frac{3^x + 1}{2}$$

$$B) \frac{5^x - 5}{3}$$

$$C) \frac{5^x - 3}{2}$$

$$D) \frac{3^x - 3}{3}$$

$$E) \frac{3^x - 2}{3}$$

$$6. f(x) = 2^{2x-1} \Rightarrow f^{-1}(x) = ?$$

$$A) \frac{\log_2 x - 1}{2}$$

$$B)$$

$$\log_2 x - 1$$

$$C) \log_2 x - 2$$

$$D) \frac{\log_2 x + 1}{2}$$

$$E) \frac{\log_2 x - 2}{3}$$

$$7. \log_2 5 = a \Rightarrow \log_5 50 = ?$$

$$A) a \quad B) \frac{a + 1}{2} \quad C) \frac{a - 1}{2} \quad D) \frac{a + 2}{a}$$

$$E) \frac{1 + 2a}{a}$$

$$8. 2^{x+1} - 2^x = 32 \Rightarrow (SS) = ?$$

$$A) \{3\} \quad B) \{5\} \quad C) \{6\} \quad D) \{7\} \quad E) \{16\}$$

$$9. \log_4 7 = x \Rightarrow \log_4 28 = ?$$

$$A) \frac{x + 1}{x}$$

$$B) \frac{1 - x}{x}$$

$$C) 1 + x$$

$$D) \frac{1}{1 + x}$$

$$E) 1 - x$$

$$10. \log_3(a - 2) = 1 \Rightarrow a = ?$$

$$A) 1$$

$$B) 2$$

$$C) 3$$

$$D) 4$$

$$E) 5$$

$$11. \log_5 3 = x \Rightarrow \log_{25} 18 = ?$$

$$A) x + 3$$

$$B) \log_5 2 + x$$

$$C) \log_5 \sqrt{2} + x$$

$$D) \log_5 \sqrt{2} - x$$

$$E) \frac{3x + 1}{3}$$

$$12. \log_a 3 + \log_a 4 = \frac{1}{2} \Rightarrow a = ?$$

$$A) 81 \quad B) 100 \quad C) 121 \quad D) 144 \quad E) 169$$

$$13. \log_{56} 8 = x$$

$$\log_{56} 7 = y \Rightarrow \log_{56} 14 = ?$$

$$A) x^2 + y^3$$

$$B) x + y$$

$$C) \frac{x + 3y}{3}$$

$$D) \frac{1}{2(x - y)}$$

$$E) 2x - y$$

$$14. \log(a + 3) + \log a = 1 \Rightarrow (SS) = ?$$

$$A) \{2\} \quad B) \{3\} \quad C) \{4\} \quad D) \left\{\frac{1}{2}\right\} \quad E) \left\{\frac{1}{3}\right\}$$



$$15. \log_2 5 \cdot \log_5 3 \cdot \log_3 1 = \log_4 (a^2 - 8) \Rightarrow a = ?$$

- A)  $\mp 2$       B)  $\mp 3$       C)  $\mp 4$       D)  $\mp 7$   
 E) 8

$$16. 2^{\log_2 x^2} + x^{\log_2 x} = 16 \Rightarrow x = ?$$

- A)  $\sqrt{2}$       B)  $\sqrt{3}$       C)  $-\sqrt{2}$       D)  $-\sqrt{5}$   
 E) 3

$$17. 5^{\log_5 (a-2)} + 6^{2 \log_6 a} = 10 \Rightarrow (SS) = ?$$

- A) {3}      B) {2}      C) {1}      D) {-2}      E)  $\emptyset$

$$18. \log_3 (x-2) + \log_3 6 = 2 \Rightarrow x = ?$$

- A)  $\frac{7}{2}$       B)  $\frac{2}{7}$       C)  $\frac{3}{4}$       D) 3      E) 7

$$19. \ln x = p \Rightarrow \log x^2 = ?$$

- A)  $p \cdot \log 2e$       B)  $2p \cdot \log^e$       C)  $\frac{p}{\ln 10}$   
 D)  $p \cdot \ln 2$   
 E)  $2p \cdot \log 2$

20.  $\ln(a \cdot b) = 4x$ ,  $\ln\left(\frac{a}{b}\right) = 4y \Rightarrow x = ?$

- A)  $e^{x+y}$     B)  $e^x$     C)  $e^y$     D)  $e^{4(x+y)}$     E)  $e^{2(x+y)}$

21.  $a = 64^{\log_2 16} \Rightarrow \log_8 a = ?$

- A) 2      B) 4      C) 6      D) 8      E) 16

22.  $\begin{cases} \log_3 30 = x \\ \log_8 30 = y \end{cases} \Rightarrow \log_{24} 30 = ?$

- A)  $\frac{x+y}{x \cdot y}$       B)  $y$       C)  $\frac{x \cdot y}{x+y}$       D)  $\frac{2 \cdot x \cdot y}{x+y}$   
 E)  $\frac{2 \cdot (x+y)}{x \cdot y}$

23.  $\log_{\frac{2}{3}} (\log_5 x) < 0 \Rightarrow (SS) = ?$

$$\text{A) } x$$

$$> 0$$

$$\text{B) } x < \frac{2}{3}$$

$$\text{C) } 0 < x < \frac{2}{3}$$

$$\text{D) } x > 5$$

$$\text{E) } 0 < x < 5$$

Answers					
1. E	2. C	3. A	4. D	5. E	6. D
7. E	8. B	9. C	10. E	11. C	12. D
13. C	14. A	15. B	16. A	17. A	18. A
19. B	20. E	21. D	22. A	23. E	